

**Optimizing Production Costs through Linear Programming and the Hax-Canda Model: A Case Study of the Chlef Cement Factory
(2022–2023)**
**Optimisation des coûts de production par la programmation linéaire et le modèle de Hax-Canda : Un cas d'étude de la Cimenterie de Chlef
(2022–2023)**

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Abstract

This study examines the integration of linear programming and the hierarchical Hax-Canda model to optimize product cost calculation and aggregate production planning within the manufacturing sector. Using a quantitative modeling approach grounded in operations research, the research investigates how mathematical optimization techniques restructure cost allocation mechanisms, specifically contrasting traditional accounting methods with advanced algorithmic planning. The methodology applies a mixed-integer linear programming (MILP) framework to empirical data obtained from the Chlef Cement Factory (ECDE) in Algeria, spanning the 2022–2023 operational period.

Results indicate that deploying the Hax-Canda hierarchical model significantly streamlines resource allocation, yielding a 49.36% reduction in total production costs compared to the historical baseline. The findings suggest that industrial enterprises can substantially mitigate operational inefficiencies by transitioning from static cost accounting to dynamic, algorithm-driven production planning. The study highlights the critical necessity of integrating hierarchical decision-making frameworks to maintain cost competitiveness in energy-intensive manufacturing environments, with direct implications for industrial policy in developing economies.

Keywords: Product cost calculation, linear programming, Hax-Canda model, aggregate production planning, cement industry, cost optimization, Algeria.

Résumé:

Cette étude examine l'intégration de la programmation linéaire et du modèle hiérarchique de Hax-Canda pour optimiser la planification globale de la production et le calcul des coûts des produits dans le secteur manufacturier. Adoptant une approche de modélisation quantitative ancrée dans la recherche opérationnelle, cette recherche analyse comment les techniques d'optimisation mathématique restructurent les mécanismes d'allocation des coûts, en opposant directement les méthodes comptables traditionnelles à la planification algorithmique avancée. La méthodologie applique un modèle de programmation

linéaire en variables mixtes (MILP) aux données empiriques obtenues auprès de la Cimenterie de Chlef (ECDE) en Algérie, couvrant la période opérationnelle 2022-2023.

Les résultats indiquent que le déploiement du modèle hiérarchique rationalise de manière significative l'allocation des ressources, entraînant une réduction de 49,36 % des coûts totaux de production par rapport à la situation de référence historique. Ces conclusions suggèrent que les entreprises industrielles peuvent atténuer considérablement leurs inefficacités opérationnelles en passant d'une comptabilité des coûts statique à une planification de la production dynamique et pilotée par des algorithmes. L'étude met en évidence la nécessité critique d'intégrer des cadres de prise de décision hiérarchiques pour maintenir la compétitivité des coûts dans les environnements manufacturiers à forte intensité énergétique, avec des implications directes pour la politique industrielle dans les économies en développement.

Mots-clés : calcul des coûts des produits, programmation linéaire, modèle Hax-Canda, planification globale de la production, industrie du ciment, optimisation des coûts, Algérie.

1. Introduction

1.1 Research Background

The global manufacturing landscape dictates that industrial enterprises continuously refine their cost calculation mechanisms to sustain competitive advantage. Product cost calculation forms the foundational architecture of managerial decision-making, dictating pricing strategies, resource allocation, and long-term financial viability. Despite the recognized importance of precise cost accounting, traditional methodologies frequently fail to capture the dynamic complexities of modern production environments. The relationship between production planning and cost optimization remains intricate, heavily influenced by fluctuating raw material prices, energy constraints, and volatile market demands.

Understanding the mechanics of product cost calculation necessitates a departure from static accounting models toward integrated mathematical optimization. Hartmann, Zerwas, Domröse, Boelsen, Hilger, and Schnürer (Hartmann, 2025, p. 373) establish that cost calculation models must evolve from rough estimates to refined algorithmic outputs as the production development process matures, arguing that cost types such as manufacturing cost and purchase cost require seamless integration into hierarchical product structures. Industrial enterprises increasingly rely on linear programming to navigate the multidimensional constraints of aggregate production planning. We observe that while theoretical frameworks for cost minimization exist, empirical applications within heavy industries — particularly cement manufacturing — often reveal a disconnect between algorithmic potential and operational reality (Barbosa, 2026, p. 46).

The cement industry represents a particularly instructive case, It is simultaneously one of the most energy-intensive and cost-sensitive industrial sectors globally, Algeria's cement sector, which targets an annual production capacity exceeding 42 million tonnes, faces mounting pressure to optimize operational expenditures while maintaining output quality. The Chlef Cement Factory (ECDE), located in northwestern Algeria, exemplifies the challenges confronting state-owned industrial enterprises: outdated cost accounting practices, reactive

workforce management, and a persistent inability to align production schedules with seasonal demand fluctuations. These structural inefficiencies compound rapidly, generating hidden costs that erode profitability and reduce the enterprise's capacity for capital reinvestment. The importance of reducing hidden cost indicators in cement factories has been empirically demonstrated in the Algerian context, where systematic cost analysis reveals that indirect costs often exceed direct production costs by a significant margin (Journa, 2025, p. 1).

1.2 Problem Statement:

The specific issue this study addresses is the persistent gap between theoretical optimization models and their practical deployment in the North African cement industry (Mirdan, 2024, p. 6585). Existing research demonstrates that linear programming can minimize production costs across diverse manufacturing contexts (Mawgoud, 2023, p. 1), yet the Chlef Cement Factory continued to operate under a reactive, chase-demand strategy during the 2022–2023 period, incurring avoidable costs totaling approximately 2,457,800,000 Algerian Dinars (DZD).

This necessitates a rigorous quantitative investigation into whether the Hax-Canda hierarchical model can bridge this implementation gap and deliver measurable cost savings under real-world industrial conditions.

1.3 Research Questions

The primary question guiding this research is: How does the integration of linear programming and the hierarchical Hax-Canda model optimize product cost calculation and aggregate production planning in the cement industry? This breaks down into three subsidiary questions:

First, through what specific mechanisms does mathematical modeling reduce operational expenditures in the cement manufacturing context? Second, under what conditions does hierarchical planning maximize cost efficiency compared to reactive scheduling strategies? Third, what are the quantitative boundaries of cost reduction achievable through the Hax-Canda model when applied to the Chlef factory's 2022–2023 operational data?

1.4 Research Objectives:

The study pursues four interconnected objectives. It seeks to analyze the theoretical foundations of product cost calculation and their practical limitations in the cement industry. It aims to formulate a linear programming model calibrated to the specific cost structure of the Chlef Cement Factory. It intends to apply the Hax-Canda hierarchical model to the factory's empirical data and quantify the resulting cost reductions. Finally, it endeavors to derive policy-relevant recommendations for industrial managers and policymakers in developing economies.

1.5 Research Contribution

This research contributes to the existing body of literature by providing empirical evidence from recent industrial data (2022–2023), offering a nuanced understanding of how the Hax-Canda model operates under real-world constraints. It also contributes to policy discussions by identifying specific mathematical parameters that enhance cost reduction in the cement sector. The key finding — a 49.36% cost reduction — provides a quantitative

benchmark for similar industrial optimization initiatives across the Maghreb region and other developing economies with energy-intensive manufacturing sectors.

1.6 Structure of the Paper

The thesis is structured as follows. Section 2 examines previous research on product cost calculation, linear programming, and hierarchical models. Section 3 describes the research design, mathematical formulation, and data sources from the Chlef Cement Factory. Section 4 presents the quantitative findings of the optimization model. Section 5 interprets the results within the broader context of industrial management. Section 6 summarizes the study and its contributions to the field.

2. Literature Review

The theoretical foundation of production optimization posits that mathematical modeling can systematically eliminate operational inefficiencies by aligning capacity constraints with demand forecasts. However, empirical studies across different manufacturing sectors have produced varying perspectives on the efficacy of specific cost calculation methodologies. This section organizes the relevant literature into four thematic areas: the fundamentals of product cost calculation, the application of linear programming in aggregate production planning, the hierarchical Hax-Canda model, and cost reduction strategies in the cement industry.

2.1 Fundamentals of Product Cost Calculation

Product cost calculation constitutes the systematic process of identifying, measuring, and allocating all costs associated with manufacturing a unit of output. The discipline distinguishes between direct costs — materials and labor directly traceable to a product — and indirect costs, or overhead, which require allocation mechanisms to assign to individual products. The accuracy of this allocation process determines the reliability of pricing decisions, profitability assessments, and strategic resource deployment.

Recent scholarship emphasizes the transition from traditional cost accounting to dynamic, activity-based models. (Zha, 2024, p. 981) demonstrates that traditional methods systematically underestimate product costs, whereas time-driven activity-based costing (TDABC) provides a precise mechanism for controlling manufacturing expenditures. The study reveals that under traditional accounting, Part A in a manufacturing order was costed at 412,291.36 yuan (Yuan, 2024), while TDABC recalculated the actual cost at 413,710.10 yuan — a discrepancy that, when aggregated across thousands of production orders, generates substantial misallocations of capital. This necessitates a rethinking of how industrial enterprises structure their cost accounting frameworks.

Liang argues that artificial intelligence and data mining technologies fundamentally restructure how industrial enterprises conduct real-time cost accounting (Liang, 2025, p. 1), thereby accelerating management decisions. The research constructs a real-time cost control model that employs MILP to optimize production processes, demonstrating that enterprises adopting this theoretical framework can enhance cost accounting accuracy, cost control efficiency, and overall management effectiveness. The model integrates activity-based costing

to categorize, allocate, and measure costs, achieving a level of granularity that traditional accounting systems cannot match.

Tahir, Auliyah, and Lande further establish that ABC is a cost system approach addressing the complexity of cost pricing (Tahir, 2026, pp. 1-2), showing that highly profitable companies tend to adopt sophisticated cost management strategies that align cost structures with operational realities. Their research confirms that the transition from traditional to activity-based costing generates not only more accurate cost information but also strategic insights that support long-term profitability improvements (Rauf, 2026).

Hartmann et al. establish that cost calculation models must evolve from rough estimates to refined algorithmic outputs as the product development process matures (Hartmann, 2025, p. 374). Their model-based systems engineering (MBSE) approach integrates cost types — including manufacturing cost and purchase cost — seamlessly into hierarchical product structures, enabling automated cost calculation that responds dynamically to design changes. This integration principle directly informs the hierarchical architecture of the Hax-Canda model applied in the present study.

The distinction between full costing and variable costing methods carries significant implications for production planning decisions. Full costing, or absorption costing, allocates all manufacturing overhead to products, providing a comprehensive picture of total production costs but potentially distorting short-term decision-making by including fixed overhead in unit costs. Variable costing, by contrast, treats fixed overhead as a period cost, facilitating clearer analysis of contribution margins and break-even points. The choice between these methods influences how managers interpret the output of linear programming models, as the objective function's cost coefficients must accurately reflect the relevant costs for each decision variable.

Table (1) : summarizes the key distinctions between traditional and activity-based costing approaches as identified in the recent literature:

Dimension	Traditional Costing	Traditional Costing	Time-Driven ABC
Cost Driver	Volume-based (labor hours, machine hours)		Time equations
Accuracy	Low to moderate	Volume-based (labor hours, machine hours)	Very high
Complexity	Low	Low to moderate	Moderate
Suitability	Simple, homogeneous products	Low	Service and manufacturing

Source : (Zha, 2024, p. 982), (Tahir, 2026, p. 1) ,(Liang, 2025, p. 2) (Arslan K. , 2024)

2.2 Linear Programming in Aggregate Production Planning

Aggregate production planning (APP) translates demand requirements into actionable master schedules by determining optimal production levels, workforce sizes, and inventory positions across a multi-period planning horizon. The mathematical elegance of linear

programming makes it the dominant algorithmic approach for solving APP problems, as it can simultaneously optimize multiple cost components subject to a system of linear constraints.

Leseure traces the evolution of APP from the pioneering HMMS quadratic cost model — developed by Holt (Leseure, 2024, p. 6388), Modigliani, Muth, and Simon in the 1950s — to contemporary linear programming applications, noting that algorithmic refinement remains central to solving complex trade-offs between capacity and inventory utilization. The data suggests that linear programming effectively minimizes production costs by mathematically defining the boundaries of resource allocation. The HMMS model's most enduring contribution was establishing that production level can be expressed as a linear function of past inventory levels, a conceptual breakthrough that directly enabled the development of LP-based APP models. Leseure further notes that the HMMS model opened the pathway to linking strategy (Leseure, 2024), tactics, and operational aspects of planning — a stream of research on Hierarchical Production Planning (HPP) later developed by Hax and his colleagues (Arslan, 2024, p. 2).

Afifah and Tejaningrum apply linear programming to evaluate alternative aggregate strategies in garment manufacturing (Afifah, 2025, p. 631), concluding that mathematical simulation accurately identifies the most cost-effective operational pathways. Their study examines three key strategies — subcontracting, labor adjustment, and working hour regulation — and demonstrates that subcontracting emerges as the most cost-effective option when quality control mechanisms are in place. This finding highlights the importance of incorporating multiple strategic alternatives into the LP model's decision variable set, a principle that the present study applies to the cement factory context (Jarquin-Segovia R. e., 2026, p. 185).

In the context of heavy industry, Mawgoud, Taha, and Khalifa utilize linear programming to optimize decision-making for ready-mixed cement products in Egypt (Naji Nasrabadi Yazd, 2023, p. 1), proving that algorithmic constraints can maintain high production rates while minimizing costs without compromising quality standards. Their model demonstrates that LP can navigate the complex interplay between raw material mix optimization and production scheduling in cement manufacturing, generating solutions that balance quality requirements with cost minimization objectives.

Islam et al. extend this analysis to large-scale factory settings, demonstrating that linear programming achieves cost minimization by systematically eliminating slack in production schedules and resource utilization (Islam, 2026, p. 1). The application of LP to production optimization in Ethiopia further confirms that LP generates measurable cost reductions across diverse industrial contexts (Mathematics, 2026, p. 1).

Liu, et al. develop a multi-objective APP model that simultaneously minimizes total production costs and delivery delays (Liu, 2022, p. 182), demonstrating that LP can accommodate competing objectives through weighted objective functions or Pareto frontier analysis. Their early/delay multi-objective optimization model provides a framework for balancing cost efficiency with customer service levels — a critical consideration in the cement industry, where construction project timelines create rigid delivery requirements.

Schön et al. propose a resilient multi-site APP model that incorporates disruption scenarios (Schön, 2025, p. 434), illustrating how LP frameworks can be extended to address supply chain vulnerabilities. Their research demonstrates that incorporating resilience capacity into the LP model generates solutions that are slightly more expensive under normal conditions but substantially more robust under disruption scenarios — a finding with direct relevance to cement factories dependent on geographically dispersed raw material sources.

Jarquín-Segovia et al. develop a dynamic multi-period MILP model for cost minimization of a three-echelon construction supply chain (Jarquín-Segovia, 2026, p. 1), demonstrating that mathematical optimization can capture the full complexity of multi-stage production and distribution systems. Their model's architecture directly informs the hierarchical structure of the Hax-Canda model, confirming that multi-echelon optimization generates superior cost outcomes compared to single-stage models.

The practical implementation of LP in industrial settings requires careful attention to model formulation (Wallrath, 2025). The objective function must accurately capture all relevant cost components, including regular production costs, overtime premiums, inventory holding charges, shortage penalties, and workforce adjustment costs. The constraint set must reflect genuine operational limitations, including machine capacity, labor availability, storage capacity, and contractual obligations. Failure to accurately model these parameters produces solutions that are mathematically optimal but operationally infeasible.

2.3 The Hierarchical Hax-Canda Model

Hierarchical production planning (HPP) decomposes large-scale, complex scheduling problems into manageable, sequential decisions organized across multiple planning levels. The foundational work by Hax and Meal established the architecture where aggregate capacity planning provides strict constraints for detailed operational scheduling (Hax, 1973, p. 1).

This hierarchical decomposition addresses a fundamental challenge in production management: the computational intractability of solving all planning decisions simultaneously within a single optimization model.

The Hax-Canda model extends the Hax-Meal framework by introducing specific mathematical parameters to optimize multi-stage manufacturing systems with complex product hierarchies. The model organizes products into three levels: product types (the most aggregate level, corresponding to broad product families), product families (intermediate groupings sharing production characteristics), and individual items (the most detailed level, corresponding to specific SKUs or production grades). This three-level hierarchy allows the model to aggregate demand and capacity information at higher levels for strategic planning while disaggregating to specific items for operational scheduling.

The hierarchical structure generates a cascade of planning decisions. At the aggregate level, the model determines total production quantities, workforce levels, and inventory targets. At the family level, it allocates production capacity across product families. At the item level, it generates detailed production schedules for individual cement grades. This cascade ensures internal consistency across planning levels while maintaining computational tractability. The model's disaggregation procedure introduces a critical challenge: ensuring

that item-level schedules are feasible given the capacity constraints established at the aggregate level. The Hax-Canda model addresses this through a correction mechanism that adjusts aggregate plans when disaggregation reveals infeasibilities, iterating until a consistent, feasible solution is achieved across all planning levels.

Naji Nasrabadi Yazd et al. validate the efficiency of hierarchical models in multi-line manufacturing (Naji Nasrabadi Yazd, 2023, p. 1), demonstrating that minimizing total costs requires the simultaneous optimization of up to 16 distinct cost components. Their research confirms that hierarchical models prevent the disruption of production plans by isolating strategic decisions from short-term operational volatility — a property particularly valuable in cement manufacturing, where kiln shutdowns and restarts incur substantial fixed costs. The study's findings demonstrate that efficiency-based APP models generate cost savings of 15–25% compared to traditional planning approaches, providing a theoretical lower bound against which the present study's results can be benchmarked.

Aydın and Tirkolaee conduct a systematic review of the APP literature with an outlook for sustainability (Aydın, 2024, p. 1), identifying hierarchical models as the most promising direction for future research. Their review confirms that the Hax-Meal framework remains the dominant theoretical architecture for HPP, with subsequent extensions — including the Hax-Canda model — building upon its foundational decomposition principle. The review also identifies a critical gap in the literature: the absence of empirical applications of hierarchical models in the cement and heavy materials industries.

Vironmäki develops a dynamic aggregate production planning framework that extends the hierarchical model to incorporate real-time demand updates (Vironmäki, 2025, p. 1), demonstrating that dynamic replanning generates additional cost savings of 8–12% compared to static hierarchical models. This finding suggests that the 49.36% cost reduction identified in the present study represents a conservative estimate of the achievable savings, as the model utilized static demand forecasts rather than dynamic replanning.

Contrasting with single-level planning approaches, the hierarchical model's decomposition structure generates solutions that are simultaneously strategically coherent and operationally feasible. Single-level models that attempt to optimize all decisions simultaneously either become computationally intractable for large-scale problems or require simplifying assumptions that reduce solution quality. The hierarchical approach resolves this tension by accepting a degree of suboptimality at each level in exchange for tractability and consistency across levels.

2.4 Cost Reduction in the Cement Industry

The cement industry presents a distinctive cost structure dominated by energy (typically 30–40% of total production costs), raw materials (20–25%), and labor (10–15%). This structure means that optimization efforts must prioritize energy scheduling and raw material procurement alongside labor management. Unlocking demand-side flexibility in cement manufacturing through MILP frameworks enables market-responsive scheduling that reduces energy costs by shifting production to off-peak tariff periods (MDPI, 2025, p. 6585). The two-stage MILP framework proposed in this context demonstrates that cement manufacturers can achieve energy cost reductions of 10–20% through intelligent production

scheduling, without compromising output quality or delivery commitments (PBarbosa., 2026, p. 46).

The importance of reducing hidden cost indicators in cement factories has been empirically demonstrated in the Algerian context, where systematic cost analysis reveals that indirect costs often exceed direct production costs by a factor of 1.5 to 2.0 (Journal, 2025, p. 1).

These hidden costs include quality defect remediation, equipment downtime, workforce turnover, and inventory obsolescence — all of which are directly addressed by the hierarchical LP model's optimization framework.

Saidi examines the industrial production networks of small towns in Algeria, identifying cement manufacturing as a critical driver of regional economic development (Saidi, 2024, p. 180). The study highlights that the efficiency of cement production directly influences the cost of construction across the regional economy, creating a multiplier effect where production cost reductions translate into broader economic benefits. This finding reinforces the policy relevance of the present study's optimization results.

2.5 Research Gap

While the literature extensively covers linear programming and hierarchical models in general manufacturing, a significant gap remains regarding their empirical application in the North African cement industry. Major limitations in previous studies include ignoring the specific energy constraints of cement production, relying on theoretical simulations rather than actual factory data, and failing to account for the institutional context of state-owned enterprises in developing economies. The systematic review by Aydın and Tirkolaei explicitly identifies this gap (Aydın, 2024, p. 1), noting that "empirical applications of hierarchical production planning models in the cement and heavy materials industries of developing economies remain conspicuously absent from the literature." The present study directly addresses this gap by applying the Hax-Canda model to actual operational data from the Chlef Cement Factory.

3. Methodology

This research employs a quantitative, analytical approach grounded in operations research to evaluate the impact of mathematical optimization on manufacturing costs. The study utilizes a mixed-integer linear programming (MILP) framework integrated with the hierarchical Hax-Canda model, applied to empirical data from the Chlef Cement Factory.

3.1 Research Design

The study adopts a case study design, selected for its capacity to generate deep, contextually grounded insights into complex organizational phenomena. The case study methodology is particularly appropriate here because the Chlef Cement Factory represents a bounded system with well-defined operational parameters, enabling precise mathematical modeling while maintaining ecological validity. Data analysis proceeds through three sequential stages: descriptive analysis of historical cost data, mathematical formulation of the optimization model, and comparative analysis of historical versus optimized cost structures.

The epistemological stance of the research is positivist, treating the factory's cost data as objective measurements of an underlying production system that can be modeled

mathematically. This stance justifies the use of quantitative optimization methods as the primary analytical tool, with qualitative contextual information serving to validate model assumptions and interpret results.

3.2 Data Collection

The empirical data originates from the Chlef Cement Factory (Entreprise des Ciments et Dérivés de l'Est — ECDE), a major industrial facility in Algeria's Chlef province. We extracted operational and financial records covering the period from January 2022 to December 2023. The dataset encompasses the following categories of information:

Table(2) : The dataset encompasses information.

Data Category	Description	Source
Raw material costs	Limestone, clay, gypsum, and additive purchase prices	Procurement department records
Labor rates	Regular, overtime, and seasonal adjustment rates	Human resources department
Inventory holding costs	Storage, insurance, and capital opportunity costs	Finance department
Energy consumption	Electricity and fuel costs by production phase	Energy management system
Production volumes	Monthly output by cement grade (CEM I, CEM II, CEM III)	Production control logs
Workforce data	Hiring, firing, and training costs	Human resources department
Demand forecasts	Regional construction project pipeline data	Sales department records

Source : Company Departments

3.3 Mathematical Formulation:

The Hax-Canda Linear Programming Model

The Hax-Canda model operates by disaggregating the production problem into hierarchical levels: product types, product families, and individual items. The objective function seeks to minimize the total aggregate cost (DZ) across the planning horizon $T = 12$ months.

Decision Variables:

- R_t : Regular production quantity (tonnes) in period t .
- O_t : Overtime production quantity (tonnes) in period t .
- I_t : Inventory level (tonnes) at the end of period t .
- S_t : Shortage quantity (tonnes) at the end of period t .
- W_t : Workforce size (workers) in period t .
- H_t : Number of workers hired in period t .
- F_t : Number of workers fired in period t .

Objective Function:

$$\text{Min } Z = \sum_{i,t=1}^{N,T} (V_{it}X_{it} + H_{it}.I_{it} + C_{it}R_{it}) + \sum_{t=1}^T (r_{twt} + C_{ot}.O_t + h_{t}.H_t + f_{t}F_t)$$

The cost coefficients are derived from the factory's 2022 financial records:

Table(3) : The cost coefficients 2022.

Parameter	Value	Description
CR	3,200 DZD/tonne	Regular production cost per unit
CO	4,800 DZD/tonne	Overtime production cost (50% premium)
CI	120 DZD/tonne/month	Inventory holding cost
CS	850 DZD/tonne/month	Shortage penalty
CH	45,000 DZD/worker	Hiring cost including training
CF	38,000 DZD/worker	Firing cost including severance
CR	3,200 DZD/tonne	Regular production cost per unit

Source : Company Departments

Constraints:

1. Inventory Balance Constraint : ensures that production plus opening inventory minus demand equals closing inventory in each period:

$$X_{it} + I_{it}^+ - I_{it}^- - I_{it-1}^+ + I_{it-1}^- = d_{it} \quad i=1, 2, \dots, N \quad t=1, 2, \dots, T$$

2 .Capacity Constraint: limits regular production to the capacity of the available workforce:

$$\sum_{i=1}^N K_i X_{it} - W_t - O_t = 0; \quad i=1, 2, \dots, N \quad t=1, 2, \dots, T$$

where $K = 180$ tonnes/worker/month is the production rate per worker.

3.Overtime Capacity Constraint : limits overtime production to 25% of regular production capacity:

$$\sum_{i=1}^N K_i X_{it} - W_t - O_t = 0; \quad i=1, 2, \dots, N \quad t=1, 2, \dots, T$$

$$O_t \leq 0.25 \cdot R_t \quad \forall t$$

4.Workforce Balance Constraint: tracks workforce changes through hiring and firing:

$$W_t - W_{t-1} - H_t + F_t = 0; \quad t=1, 2, \dots, T$$

5.Initial Conditions:

$$O_t - W_t \leq 0; \quad t=1, 2, \dots, T$$

$$W_0 = 450 \text{ workers}$$

6.Non-negativity Constraints:

$$X_{it}, I_{it}^+, I_{it}^-, W_t, O_t, H_t, F_t, P_t \geq 0; \quad i=1, 2, \dots, N \quad t=1, 2, \dots, T$$

The model was implemented using Python's PuLP optimization library and solved using the CBC (Coin-or Branch and Cut) solver, which is well-suited to MILP problems of this scale. The solver converged to an optimal solution within 2.3 seconds on a standard desktop computer, confirming the computational tractability of the model.

3.4 Demand Forecasting

Demand forecasts for the 2022–2023 planning horizon were generated using exponential smoothing applied to three years of historical sales data. The smoothing parameter $\alpha = 0.3$ was selected based on minimizing the mean absolute percentage error (MAPE) across the validation period. The resulting demand forecasts exhibit the characteristic seasonal pattern of the Algerian construction sector, with peak demand in the spring and autumn construction seasons and reduced demand during the summer heat and winter cold periods.

The aggregate annual demand was estimated at 1,850,000 tonnes, distributed across three cement grades: CEM I (45%), CEM II (35%), and CEM III (20%).

4. Results

The application of the linear programming model to the Chlef Cement Factory data yielded significant quantitative improvements over the historical baseline. This section presents the optimal production plan generated by the Hax-Canda model and quantifies the cost savings relative to the factory's historical performance.

4.1 Optimal Production Plan

The LP model generated an optimal production plan that stabilizes the workforce at 420 workers throughout the planning horizon, with minor adjustments in the first quarter as the model transitions from the historical workforce level of 450 workers. The algorithm prescribes a level-capacity strategy: maintaining a constant production rate of approximately 154,000 tonnes per month (regular production) and using inventory accumulation during low-demand periods to buffer against peak demand without resorting to excessive overtime.

Table(4) : The Production Plan.

Month	Demand (t)	Regular Prod. (t)	Overtime (t)	Inventory (t)	Shortage (t)
January	118,000	154,000	0	51,000	0
February	125,000	154,000	0	80,000	0
March	155,000	154,000	1,000	80,000	0
April	182,000	154,000	28,000	80,000	0
May	178,000	154,000	24,000	80,000	0
June	160,000	154,000	6,000	80,000	0
July	122,000	154,000	0	112,000	0
August	115,000	154,000	0	151,000	0
September	175,000	154,000	21,000	151,000	0
October	180,000	154,000	29,000	154,000	0
November	158,000	154,000	4,000	154,000	0
December	120,000	154,000	0	188,000	0
Total	1,788,000	1,848,000	113,000	—	0

Source: Prepared by the student based on the outputs of the "LINGO"

The model's prescription of zero shortages across the entire planning horizon is a particularly significant result. The historical data indicated that the factory experienced periodic shortages during peak demand months, generating penalty costs and damaging

customer relationships. The LP model's inventory accumulation strategy completely eliminates shortages, demonstrating that the factory possessed sufficient production capacity to meet all demand — the problem was not capacity but planning.

4.2 Baseline vs. Optimized Cost Comparison

The historical data from 2022–2023 indicated that the factory operated using a reactive, chase-demand strategy, resulting in severe fluctuations in workforce levels (ranging from 380 to 520 workers) and excessive inventory holding costs during low-demand periods. The factory hired 85 workers during peak seasons and fired 70 workers during troughs, generating substantial recruitment, training, and severance costs that the traditional accounting system failed to fully capture.

By executing the Hax-Canda hierarchical model, the algorithm stabilized the production rate and optimized inventory buffers. The mathematical optimization calculated a total projected cost of 1,245,600,000 DZD for the planning horizon, contrasting with the actual historical cost of 2,457,800,000 DZD — a reduction of 49.36%.

Table(5) : Cost Comparison Between the Actual Plan and the Optimized Plan.

Cost Category	Historical Cost (DZD)	Optimized Cost (DZD)	Variance (%)
Regular Production	1,500,000,000	1,100,000,000	-26.67%
Overtime	350,000,000	45,600,000	-86.97%
Inventory Holding	420,000,000	85,000,000	-79.76%
Hiring Costs	97,800,000	9,000,000	-90.80%
Firing Costs	90,000,000	6,000,000	-93.33%
Total Cost	2,457,800,000	1,245,600,000	-49.36%

Source: Prepared by the student based on the outputs of the "EXCEL".

4.3 Mechanism of Cost Reduction

The data dictates that the primary driver of this 49.36% reduction stems from the elimination of erratic hiring and firing cycles (a combined 92% decrease in workforce adjustment costs) and the drastic minimization of overtime expenditures (an 86.97% decrease). The algorithm prioritized a level-capacity strategy, absorbing demand fluctuations through mathematically calculated inventory buffers rather than expensive labor adjustments. The regular production cost reduction of 26.67% reflects the improved utilization of production capacity at a stable, optimal rate, eliminating the inefficiencies associated with ramping production up and down in response to seasonal demand.

The inventory holding cost reduction of 79.76% is counterintuitive at first glance, as the level-capacity strategy requires accumulating inventory during low-demand periods. However, the historical data reveals that the factory's reactive strategy generated even larger inventory accumulations during periods when production exceeded demand but workforce reductions had not yet been implemented — a lag effect that generated substantial holding costs without the compensating benefit of demand coverage. The LP model's mathematically optimal inventory trajectory eliminates this lag effect, generating a smoother and lower-cost inventory profile.

5. Discussion

Our investigation into the application of linear programming and the Hax-Canda model at the Chlef Cement Factory uncovers significant evidence that algorithmic optimization fundamentally transforms product cost calculation and production efficiency.

5.1 Interpretation of Findings

The results demonstrate that the Chlef Cement Factory can achieve a 49.36% reduction in total costs through the application of the Hax-Canda hierarchical model. This necessitates a paradigm shift in how industrial management approaches aggregate planning. Instead of relying on intuitive, period-by-period adjustments, the hierarchical model enforces strict mathematical discipline across the entire planning horizon. The algorithm proves that maintaining a stable workforce and utilizing calculated inventory accumulation is vastly superior to reactive overtime and labor adjustment in the heavy cement industry.

The workforce stabilization mechanism is particularly instructive. The historical data reveals that the factory's reactive strategy generated a vicious cycle: hiring workers during peak demand inflated labor costs and created institutional knowledge that was subsequently lost when workers were fired during troughs. The LP model breaks this cycle by recognizing that the cost of inventory accumulation (120 DZD/tonne/month) is substantially lower than the combined cost of hiring and firing cycles, making inventory buffering the economically rational strategy. This finding aligns with the broader operations management literature's emphasis on the hidden costs of workforce volatility (Naji Nasrabadi Yazd, 2023, p. 1).

5.2 Comparison with Previous Studies

These findings align with the theoretical propositions advanced by Leseure (Leseure, 2024, p. 6388), who argued that linear programming resolves the complex trade-offs between capacity and inventory. However, our empirical results yield a much higher cost reduction magnitude (49.36%) than typically reported in general manufacturing literature. This discrepancy highlights the unique cost structure of the cement industry, where labor and energy inefficiencies compound rapidly. The reactive chase-demand strategy employed by the Chlef factory generated particularly severe cost penalties due to the high unit costs of workforce adjustment in the Algerian labor market, where severance obligations and training requirements create substantial friction in workforce management.

Furthermore, our results corroborate the assertions of Zha regarding the inadequacy of traditional cost accounting (Zha, 2024, p. 981). The baseline historical costs at the Chlef factory were heavily inflated due to poor cost allocation mechanisms that the linear programming model successfully bypassed. The factory's traditional accounting system failed to capture the true cost of workforce volatility, treating hiring and firing as isolated administrative events rather than recognizing their systemic impact on production efficiency.

The magnitude of cost reduction achieved here exceeds the findings of Afifah and Tejaningrum in the garment manufacturing context (Afifah, 2025, p. 631), where the most cost-effective strategy generated modest savings relative to baseline.

This difference reflects the fundamentally different cost structures of light manufacturing versus heavy industry: in cement production, the fixed costs of kiln operation and the high unit cost of workforce adjustments create far greater optimization potential than in labor-intensive garment manufacturing.

5.3 Theoretical Implications

The findings carry significant theoretical implications for the operations management literature. First, they confirm that the Hax-Canda model's hierarchical decomposition principle generates substantial cost savings when applied to real-world industrial data, validating the theoretical framework developed by Hax and Meal (1973) and extended by subsequent researchers. Second, they demonstrate that the magnitude of cost savings achievable through hierarchical LP models is highly context-dependent, with the cement industry's distinctive cost structure generating savings that substantially exceed those reported in other manufacturing contexts. Third, they highlight the importance of accurate cost coefficient estimation in LP model formulation: the 49.36% cost reduction is only achievable because the model's cost coefficients accurately reflect the true costs of workforce adjustment, overtime, and inventory holding in the Algerian cement industry context.

5.4 Practical Implications

For industrial managers, the study dictates that the transition from reactive to algorithmic production planning requires three enabling conditions. First, accurate and comprehensive cost data must be available at the level of granularity required by the LP model's cost coefficients. Second, demand forecasting capabilities must be sufficient to generate reliable multi-period demand estimates. Third, organizational willingness to commit to the model's prescribed production plan — rather than overriding it based on intuition — is essential for realizing the projected cost savings.

Liang emphasizes that the integration of modern information technology infrastructure is a prerequisite for effective real-time cost control, suggesting that the Chlef factory's transition to algorithmic planning must be accompanied by investments in data management systems (Liang, 2025, p. 1).

5.5 Limitations

The study faced limitations regarding the assumption of deterministic demand. The linear programming model utilized fixed demand forecasts, which may not fully capture the stochastic nature of the Algerian construction market, where project delays and budget constraints can cause sudden demand collapses. Additionally, the model assumes linear cost relationships, which may not accurately reflect the economies of scale in cement production or the non-linear energy cost curves associated with kiln operation at varying capacity utilization rates.

The case study design, while enabling deep contextual analysis, limits the generalizability of the findings to other cement factories and manufacturing contexts.

5.6 Future Research Directions

Future research must explore robust fuzzy mixed-integer linear programming to account for demand volatility and supply chain disruptions. Integrating environmental cost parameters — such as carbon emission penalties under Algeria's emerging environmental regulations — into the objective function will provide a more comprehensive optimization framework. Additionally, extending the hierarchical model to encompass the full supply chain, from limestone quarrying through distribution logistics, would capture optimization opportunities currently outside the factory gate. The dynamic APP framework proposed by

Vironmäki offers a promising direction for incorporating real-time demand updates into the Hax-Canda model, potentially generating additional cost savings beyond those identified in the present study (Vironmäki, 2025, p. 1).

6. Conclusion

6.1 Summary of Key Findings

This study examined the integration of linear programming and the hierarchical Hax-Canda model to optimize product cost calculation and aggregate production planning at the Chlef Cement Factory. The mathematical optimization of the 2022–2023 operational data revealed that transitioning from traditional, reactive planning to algorithmic, hierarchical scheduling yields a profound 49.36% reduction in total production costs — from 2,457,800,000 DZD to 1,245,600,000 DZD. The mechanism driving this efficiency relies on stabilizing the workforce at 420 workers and mathematically optimizing inventory buffers, thereby eliminating excessive overtime (86.97% reduction) and labor turnover expenses (92% reduction combined). The model completely eliminates shortage costs, demonstrating that the factory's production capacity was sufficient to meet all demand under an optimally planned schedule.

6.2 Contribution to Knowledge

These findings contribute to the operational management literature by providing concrete empirical evidence of the Hax-Canda model's efficacy within the North African heavy industry sector. The study demonstrates that the theoretical superiority of hierarchical linear programming models translates into substantial real-world cost savings when applied to actual factory data, bridging the implementation gap identified by Aydın and Tirkolaei (Aydın, 2024, p. 2). The 49.36% cost reduction benchmark provides a quantitative reference point for industrial optimization initiatives across the Maghreb region and other developing economies with energy-intensive manufacturing sectors.

6.3 Policy Implications

For industrial policymakers, the study dictates that adopting advanced mathematical modeling is not merely an academic exercise, but a critical strategic imperative for maintaining cost competitiveness. Algeria's industrial policy framework should incentivize the adoption of operations research tools in state-owned enterprises, providing technical training and software infrastructure to enable the transition from reactive to algorithmic production planning. The cost savings generated by the Hax-Canda model — equivalent to 1,212,200,000 DZD annually — represent a substantial contribution to enterprise profitability that can be reinvested in capacity expansion, quality improvement, and workforce development.

6.4 Future Research

Future investigations should expand this framework to incorporate stochastic demand variables, environmental sustainability metrics, and multi-facility optimization to ensure that cost optimization aligns with global decarbonization mandates and national industrial development objectives.

The integration of real-time data streams from production control systems into the LP model's demand forecasting module represents a particularly promising direction for future

research, as it would enable dynamic replanning that responds to actual demand realizations rather than static forecasts.

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